Dynamic Heckscher-Ohlin Model under Monopolistic Competition

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Contents

I Introduction

A relationship between international trade and economic growth has been one of the important problems for economics. Even in recent years, there are many empirical studies on the correlation between economic growth and the amount of trade as a fact in economic growth. For example, Harrison (1996), Rodriguez and Rodrik (2000), Lee et al. (2004) argue that the relation between openness and growth is still an open question. However, what kind of mechanism works between trade and growth has not been considered for a long time.

We consider a growth model with international trade and monopolistic competition in our analyses. From a viewpoint of growth theory, our growth model is greatly influenced from Grossman and Helpman (1991) and Romer (1986). We introduce monopolistic competition as a imperfection of market into our model. On the other hand, we consider small open economy with two kinds of primary factor of production ; labor and human capital. Therefore, our model can be seen as a dynamic version of Heckscher-Ohlin model. Therefore, the famous results of Heckscher-Ohlin model are considered in dynamic model.

Our analysis closely resembles those of Ciccone and Matsuyama (1996,1999). As same as Ciccone and Matsuyama (1996, 1999), we also find a multiplicity of steady states and a kind of indeterminacy[1] in our analysis. However, the mechanism of our model is quite different from that of Ciccone and Matsuyama (1996, 1999). We consider two kinds of primary factor of production, that is, labor and human capital. Substitutability of the two inputs mainly causes the multiplicity of steady states[2], In this paper, a steady state with larger vatiety of intermediate goods is called upper steady state. On the contrary, a steady state with smaller variety is called lower steady state.

Moreover, we consider how scale of the economy, prices of goods, and the cost of R & D influence the steady states. Multiplicity of steady states provides contrary conclusions of comparative statics between upper and lower steady states. For example, "an" increase in human capital raises the variety of upper steady state. On the other hand, that decreases the variety of lower steady state. In this case, we confirm that an usual result of Rybczynski Theorem only at the upper steady state. This means that the economy tends to be more effective at production of good that is intensive in human capital.

The remaining of this paper is organized as follows. In the section 2, we describe the model. In the section 3, we specify the dynamical system and explore the equilibrium path. In the section 4, we consider how the parameters of model affect the steady states and discuss some implication of economic policy.

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[1] Indeterminacy is an important topic of economic dynamics. Especially, dynamic models with this indeterminacy are applied to business cycle theory and other macroe-conomic phenomena. See Benhabib and Farmer (1994) and Benhabib and Perli (1994). Greiner and Semmler (1996) considered indeterminacy problem in an endogenous growth model.

[2] Ikeshita(2002) shows the possibility of multiple steady states when production factor is only labor and international trade exists.

II The Model

In this section, we set up the model that is considered in this paper. We assume that the economy is small and trades with other foreign countries. For simplicity, we assume that there is no capital transfer, then, investment in this economy is financed by domestic saving.

1 The Behavior of Consumer

In this economy, representative consumer earns wage by supplying his labor and human capital. He also receives the interest from his assets. We assume that consumer supplies L units of labor and H units of human capital to production sectors. Consumer also trades two kinds of consumption goods, good 1 and good 2 with other country. He decides a sequence of consumption to maximize his lifetime utility subject to the intertemporal budget constraint. The lifetime utility is given by[3]

$$U(t) = \int_t^\infty e^{-\rho(\tau-t)} \log[C_1(\tau)]^\beta [C_2(\tau)]^{1-\beta} d\tau, \tag{1}$$

where ρ is subjective discount rate. $C_i(t)$ (i=1,2) is consumption of good i at time t. $\log[C_1(t)]^\beta[C_2(t)]^{1-\beta}$ is flow of utility at time t and β is parameter of preference. Consumer confronts following intertemporal budget constraint.

$$\int_t^\infty e^{-\int_t^\tau r(s)ds} E(\tau)d\tau \leq \int_t^\infty e^{-\int_t^\tau r(s)ds} [w(\tau)L + w_H(\tau)H]d\tau + W(t). \tag{2}$$

$E(t)$ is the expenditure of consumer at time t, $w(t)$ is wage rate of labor, $w_H(t)$ is wage rate of human capital. $r(t)$ is interest rate and $W(t)$ is the value of asset holding. We can decompose this maximization problem to two stages. The first problem is an usual static problem, that is, to maximize $[C_1(t)]^\beta[C_2(t)]^{1-\beta}$ subject to $q_1C_1(t)+q_2C_2(t) \leq E(t)$. By solving this problem, we derive an indirect utility as a linear function of $E(t)$. Second problem is how consumer allocates his income to expenditure or saving. The solution of this maximization problem

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[3] In this paper, (t) denotes the variable at period t.

is given by usual Euler equation.

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho\ . \tag{3}$$

At the same time, consumer's budget constraint is binding, that is,

$$\int_t^{\infty} e^{-\int_t^{\tau} r(s)ds}(E(\tau) - w(\tau)L - w_H(\tau)H)d\tau = W(t). \tag{4}$$

2 Firm

Next we formulate the behavior of firm. There are two kinds of consumption goods and these goods are traded between this country and foreign country in this paper. In the context of traditional Heckscher-Ohlin framework, both countries produce these two goods if the prices of these goods and factor endowment are in the proper circumstance. On the other hand, The countries may specialize one good if factor endowment is strongly biased. However analysis of the pattern of production is important when we want to capture the complete behavior of the economy, it is too complicated because we now consider the dynamic setting. Hence, we analyze the "diversification" case only. We begin with the consumption goods sector. The consumption goods are produced by perfectly competitive firms and the production function of good 1 is given by $Y_1(t)=X(t)^{\alpha}H_1(t)^{1-\alpha}$. X(t) is defined as

$$X(t) = \left(\int_0^{n(t)} [x(j,t)]^{1-1/\sigma} dj\right)^{\sigma/(\sigma-1)}, \quad \sigma > 1, \tag{5}$$

where $Y_1(t)$ is the quantity of good 1 produced at time t and $H_1(t)$ is the quantity of human capital used for production of good 1. α is a parameter of production function. $n(t)$ is a number of intermediate goods that is available for production[4]. $x(j,t)$ is quantity of intermediate good j for production of good 1. We assume that $\sigma>1$, which causes increasing return.

[4] For simplicity, $n(t)$ is not discrete, but continuous.

On the other hand, good 2 is produced with labor only, that is, $Y_2(t)=L_2(t)$,where $L_2(t)$ is quantity of labor used for production of good 2. We derive the demand function of intermediate good j as

$$x(j,t) = \frac{\alpha q_1 Y_1(t)}{\int_0^{n(t)} p(j,t)^{1-\sigma} dj} p(j,t)^{-\sigma}. \tag{6}$$

Similarly, the demand for human capital is

$$H_1(t) = (1-\alpha)\frac{q_1 Y_1(t)}{w_H(t)}. \tag{7}$$

In the intermediate goods sector, each firm produces some kind of intermediate goods. The know-how of producing intermediate goods is invented by the efforts in the R & D sector. To produce one unit of intermediate goods, all firm needs one unit of labor. Because producers of intermediate goods set the monopoly prices, all of the firm set the same price $p(t)$ which is given by

$$p(j,t) = p(t) = \left(1 - \frac{1}{\sigma}\right)^{-1} w(t). \tag{8}$$

Let us define a new price index $P=(\int_0^n [p(j)]^{1-\sigma} di)^{1/(\sigma-1)}$, which implies a proper price of $X(t)$. From (8), we can derive

$$\frac{P(t)}{w(t)} = \left(1 - \frac{1}{\sigma}\right)^{-1} n(t)^{\frac{1}{1-\sigma}}. \tag{9}$$

Equation (9) implies that the price index of intermediate goods declines relatives to wage when the variety of intermediate goods $n(t)$ increases. Moreover, because all prices of intermediate goods are same, the demands of them are also same, that is, $x(j,t)=x(t)$. From (5),

$$X(t) = x(t)n(t)[n(t)]^{1/(\sigma-1)}, \tag{10}$$

where $x(t)n(t)$ is the quantity of labor used for production of intermediate goods. Therefore, even if the quantity of labor does not change, $X(t)$ increases when the variety increases. We also obtain the relation between wage of human capital and price index of intermediate goods, that is given by

$$\frac{w_H(t)}{P(t)} = \frac{1-\alpha}{\alpha}\frac{X(t)}{H_1(t)}. \tag{11}$$

From (11), we can derive

$$\frac{w_H(t)}{P(t)} = \frac{w_H(t)}{w(t)}\left(1-\frac{1}{\sigma}\right)n(t)^{\frac{1}{\sigma-1}}. \tag{12}$$

Because we are interested in the case of diversification, the unit costs of goods must be equal to the prices, which implies

$$Z[w(t)]^{\alpha}[w_H(t)]^{1-\alpha}n(t)^{\frac{\alpha}{1-\sigma}} = q_1, \tag{13}$$

where

$$Z = \left[\left(1-\frac{1}{\sigma}\right)\frac{\alpha}{1-\alpha}\right]^{-\alpha}\frac{1}{1-\alpha}.$$

Similarly, same relation is satisfied for good 2,

$$w(t) = q_2. \tag{14}$$

From (13) and (14), relative factor price depends on prices of goods and variety of intermediate goods. When we define ω is the relative factor price, ω is given by

$$\frac{w_H(t)}{w(t)} = \left[\frac{1}{Z}\frac{q_1}{q_2}n(t)^{\frac{\alpha}{\sigma-1}}\right]^{\frac{1}{1-\alpha}} = \omega(n(t)). \tag{15}$$

Therefore, from (12), wage rate of human capital and price index is derived as

$$\frac{w_H(t)}{P(t)} = \left(1-\frac{1}{\sigma}\right)n(t)^{-\frac{1}{1-\sigma}}\omega(n(t)). \tag{16}$$

At the end of this section, we consider the profit earned by producers of intermediate goods, $\pi(t)$ is profit of each firm. Total value paid to intermediate sector is $P(t)X(t) = \alpha q_1 Y_1(t)$. Moreover, intermediate sector earns profit at the fraction $1/\sigma$ of its revenue and the rest is paid to labor, then total profit is $\alpha q_1 Y_1(t)/\sigma$. Hence, the profit per firm is given by

$$\pi(t) = \frac{\alpha q_1 Y_1(t)}{\sigma n(t)}. \tag{17}$$

3 R & D Sector

R & D sector is in the circumstance of free entry, that is, every firms can enter or exit this sector. If firms succeed to R & D activities, they enter the intermediate goods markets and receive their profit monopolistically. On the other hand, a_R units of human capital is used to develop one kind of blueprint.

Next we consider the value of developing the blueprint. Firms that succeed to R & D activity continue to achieve the profit by supplying intermediate goods. Therefore, the value of performing R & D is given by

$$v(t) = \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds} \pi(\tau) d\tau. \tag{18}$$

We interpret $v(t)$ as the value of holding this firms. By differentiating $v(t)$, we obtain the following no-arbitrage condition, that is

$$r(t)v(t) = \pi(t) + \dot{v}(t). \tag{19}$$

RHS of (19) is all of the return of holding this firm. On the other hand, LHS of that implies the return of devoting money to riskless bonds, that is, these two returns are same in the equilibrium of capital market.

a_R units of human capital is needed to develop one unit of blueprint. In other words, a_R units of human capital provides the value $v(t)$. The amounts of labor devoted to R & D sector is determined by the following condition,

$$(w_H(t)a_R - v(t))\dot{n}(t) = 0 \quad \dot{n} \geq 0. \tag{20}$$

If $\dot{n}(t)>0$, $w_H(t)a_R=v(t)$ must be satisfied in equilibrium.

4 Resource Constraints

In this subsection, we consider constraints of labor and human capital. Before proceeding, we define a new variable. u_H is a fraction of human capital devoted to the manufacturing sector of good 1, which implies $H_1(t)=u_H(t)H$. Moreover, human capital is devoted to production of good 1 or performing R & D. This implies that

$$H - a_R\dot{n}(t) = u_H(t)H. \tag{21}$$

As same as human capital, u_L denotes a fraction of labor devoted to the production of good 2, that is, $L_2(t)=u_L(t)L$. Labor is used in only two sectors ; the production of good 2 and the production of intermediate goods, which means

$$x(t)n(t) = (1 - u_L(t))L. \tag{22}$$

From above relations, (10), (11), and (16),

$$\left(1 - \frac{1}{\sigma}\right) n(t)^{-\frac{1}{1-\sigma}}\omega(n(t)) = \frac{1-\alpha}{\alpha}\frac{(1 - u_L(t))Ln(t)^{\frac{1}{\sigma-1}}}{u_H(t)H}. \tag{23}$$

By making use of $q_1Y_1=E(t)$, we derive

$$w_H(t)H + w(t)L = w_H(t)a_R\dot{n}(t) + E(t) - n(t)\pi(t). \tag{24}$$

Moreover, by integrating (24),

$$\lim_{T\to\infty} n(T)v(T)e^{-\int_t^T r(s)ds} = \int_t^\infty e^{-\int_t^T r(s)ds}(w(\tau)L+w_H(\tau)H-E(\tau))d\tau+n(t)v(t). \tag{25}$$

$W(t)$, which is amount of asset held by the representative individual, satisfies $W(t)=n(t)v(t)$ from the definition. By using (4),

$$\lim_{T\to\infty} n(T)v(T)e^{-\int_t^T r(s)ds} = 0. \tag{26}$$

Ⅲ　Dynamic Behavior of the Economy

In this section, we attempt to reduce our model to a system of two differential equations. To begin with, we define a new variable V(t) as $V(t)= \frac{v(t)}{E(t)}$. Using $V(t)$, a behavior of $n(t)$ is represented by

$$\dot{n}(t) = \max\left\{\frac{H}{a_R} + \frac{L}{\bar{A}a_R\omega(n(t))} - \frac{1}{\bar{A}V(t)}, 0\right\}, \tag{27}$$

where $\bar{A} = 1+\frac{\alpha}{\sigma(1-\alpha)}$. Similarly, a behavior of $V(t)$ is derived as

$$\dot{V}(t) = \rho V(t) - \frac{1}{n(t)} J\left(1 - \frac{LV}{a_R\omega(n(t))}\right), \tag{28}$$

where $J = 1+\frac{\alpha}{\bar{A}(1-\alpha)}$. Finally, a following condition must be satisfied on the equilibrium path, that is,

$$\lim_{t\to\infty} V(t)n(t)e^{-\rho t} = 0. \tag{29}$$

The equilibrium path is characterized by (27), (28), and (29). We use this dynamical system in order to clarify the dynamic behavior of the economy. We begin with the behavior of the variety $n(t)$, From (27), by setting $\dot{n}(t)$=0, we obtain

$$V = \frac{a_R\omega(n)}{H\bar{A}\omega(n) + L}, \tag{30}$$

which is a relationship between V and n. Next, we investigate the shape of $\dot{n}$=0-curve. To derive the slope of this curve, we differentiate (30) with respect to n and we obtain

$$\frac{dV}{dn} = \frac{La_R\frac{d\omega(n)}{dn}}{(H\bar{A}\omega(n) + L)^2} > 0, \tag{31}$$

which implies that the slope of the curve is positive. $\dot{n}$=0-curve is depicted in

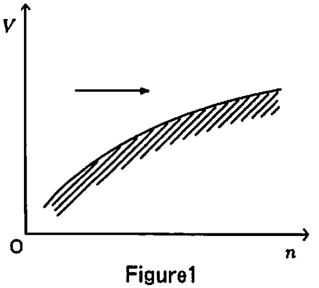

Figure1

The region below this curve is the area that satisfies $\dot{n} = 0$. Before proceeding, we consider the shape of $\dot{n} = 0$-line. We initially suppose that the economy is located on the $\dot{n} = 0$-line and the value of $V(t)$ rises and $n(t)$ does not change. From the definition of $V(t)$, this means that the expenditure decreases, therefore, the increase of $V(t)$ enhances the amount of investment and $n(t)$ become positive. To place the economy on the $\dot{n}=0$-line again, $n(t)$ must increase. The reason is that the increase in n raises the wage of human capital and restrain the R & D activities. Therefore, $\dot{n}=0$-line is depicted as upward-sloping in the Figure 1.

On the other hand, from (28), $\dot{V}(t)=0$ implies that

$$V=\frac{J}{\rho n+\frac{L}{a_R\omega(n)}}, \tag{32}$$

which is also derived as the relation between V and n. Next, we consider the shape of $\dot{V}$=0-line. By differentiating (32) with respect to n,

$$\frac{dV}{dn}=-\frac{J}{(\rho n+\frac{L}{a_R\omega(n)})^2}\frac{1}{\omega(n)}\left[\rho\omega(n)-\frac{L}{a_R}\frac{\alpha}{(\sigma-1)(1-\alpha)}\frac{1}{n}\right]. \tag{33}$$

The graphs of $\rho\omega(n)$ and $\frac{L}{a_R}\frac{\alpha}{(\sigma-1)(1-\alpha)}\frac{1}{n}$ are depicted in Figure 2. The former is a monotonically increasing function of n. The latter is a monotonically decreasing function of n and There exists the value of $\bar{n}$ that two curves intersect each other. If $n<\bar{n}$, $\frac{dV}{dn}>0$ is realized. On the other hand, if $n>\bar{n}$, $\frac{dV}{dn}$ <0. As a consequence, the combination of n and V which satisfies $\dot{V}=0$ is depicted as inversed U-shaped curve. The $\dot{V}$=0-line is pictured in Figure 3.

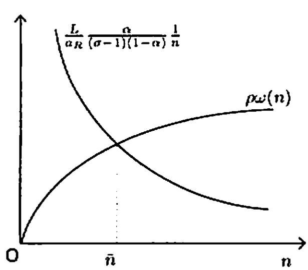

Figure2

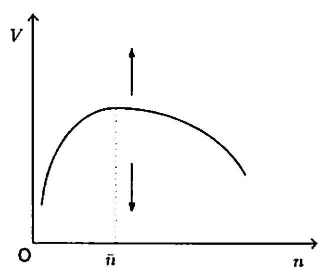

Figure3

Similar to case of $\dot{n}=0$-line, we consider the reason why the $\dot{V}=0$-curve is depicted as inversed U-shaped curve. To proceeding this analysis, it may be tractable to transform the differential equation for $\dot{V}$ as follow.

$$\dot{V} = V(t)\left(\rho - \frac{\pi(t)}{v(t)}\right). \tag{34}$$

We can interpret $\pi(t)/v(t)$ as profit rate. (34) implies that V may decline when the profit rate is larger than the discount rate. Hence, it is useful to consider the change of profit rate with n when we fix the value of V. By making use of (17), (20) and (42), the profit rate is given by

$$\frac{\pi}{v} = \frac{\sigma\alpha(1-\alpha)}{a_H}\frac{u_H H}{n}. \tag{35}$$

Moreover, $u_H H$ is a increasing function of n as same as the case of $\dot{n} = 0$-line. When V is constant, the increase in n restrains R & D activity. Hence, human capital devoted to the manufacturing of good 1, $u_H H$ increases.

From this discussion, the increase of n has two effects on profit rate. The first one by increase of variety is to restrain the profit per firm. The second is that the amounts of human capital used for the production of good 1 rise and profit per firm also increases. However the former effect provides a decrease of profit rate, the latter is positive effect on the profit rate.

Let us return to the analysis. We assume that the economy located on the $\dot{V} = 0$-line initially. When n increases and V does not change, the profit rate increases if human capital employed in good 1 production rises enough. Therefore, from (48), V must increase and restrain human capital flow to good 1 production in order to place the economy on the $\dot{V} = 0$-line again. As a result, $\dot{V} = 0$-line slopes upward. On the contrary, the profit rate goes down when the profit reduction effect is strong enough. Hence, V must be reduced and the flow of human capital to production of good 1 is strengthen in order to place the economy on $\dot{V} = 0$-line again. Consequently, $\dot{V} = 0$-curve slopes downward. Indeed, by differentiating (35) with respect to n, we can confirm that expanding production effect dominates when n is small enough. On the other hand, profit reduction effect dominates with larger n. Therefore, $\dot{V} = 0$-curve is depicted as inversed U-shaped in Figure 3.

Next, we consider the locations of $\dot{n} = 0$-line and $\dot{V} = 0$-line. By using (30) and (32), we can derive

$$\frac{a_R\omega(n)}{H\bar{A}\omega(n)+L}-\frac{J}{\rho n+\frac{L}{a_R\omega(n)}}=\frac{-a_R(\omega(n))^2}{(H\bar{A}\omega(n)+L)(\rho a_R n\omega(n))}\left[-\rho a_R n+J\bar{A}H-\frac{(1-J)L}{\omega(n)}\right], \tag{36}$$

where $\rho a_R n+J\bar{A}H$ is a declining line for n. We easily show $0<1-J<1$. On the other hand, $\frac{(1-J)L}{\omega(n)}$ is a curve whose shape is like a hyperbola. The intersection of these two curves corresponds to the point that $\dot{n}=0$-line and $\dot{V}=0$ intersect.

$\frac{(1-J)L}{\omega(n)}$ and $\rho a_R n+J\bar{A}H$ intersect in the Figure 4(a) and there are two values of n that two curves intersect. n_1^* denotes the smaller variety at which two curves intersect. On the other hand, n_2^* denotes the greater value.

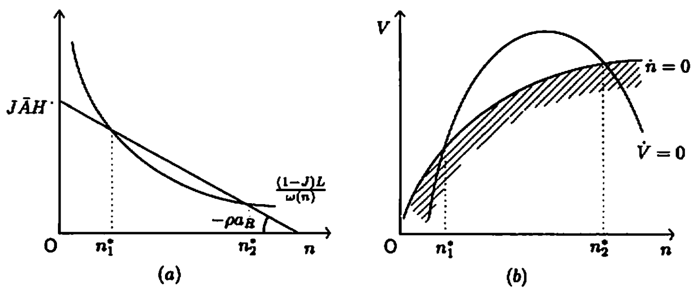

Figure4

Figure 5 is a phase diagram based on Figure 4. It is clear that equilibrium paths have some interesting characteristics. The model has two intersections, n_1^* and n_2^*, which are steady states of this model. We call S_1^* lower steady state. On the other hand, we call S_2^* upper steady state. Moreover, there are saddle paths that converge to each steady state.

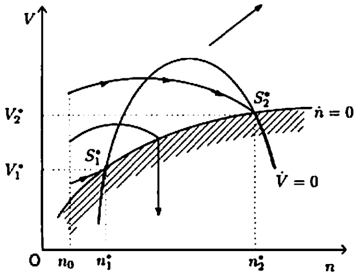

Figure5

In particular, there exist two equilibrium paths when initial value n_0 is small enough. One is a path that converges to the upper steady state and the other converges to the lower steady state. The structure of the model cannot determine which path is chosen. This implies that there is a kind of indeterminacy because of the multiplicity of steady states.

We confirm that the equilibrium path that converges to the upper steady state describes a inversed U-shaped trajectory. This result suggests that consumer initially reduces his expenditure and invests to the R & D activity. Later, he gradually increases his expenditure[5].

On the other hand, there also exists a equilibrium path that converges to lower steady state. In such a trajectory, consumer initially chooses more consumption and less investment. Finally he achieves smaller variety in the steady state.

When the initial variety is larger than n_1^*, the economy remains on the $\dot{V}=0$-line. V and n do not change anymore. No other path cannot be achieved in the equilibrium.

[5] From definition of V, we already know $v=w_H a_R$ as long as resources are used for R & D activity. Moreover factor price is determined by price of goods and the variety n because we have assumed that the economy is in the circumstance of diversification. From (15), we derive $w_H=q_2\,\omega(n)$, hence, the expenditure E decreases when both V and n increase simultaneously.

Ⅳ Comparative Statics

In this section, we observe that how the steady state value of variety, n^* and V^* are influenced by scale of economy, prices of goods. Especially, it is interesting for us to study the changes of two steady states[6].

To begin with, we examine how increase of human capital affects the steady states. Before proceeding, linear approximation of $\dot{n}=0$-line and $\dot{V}=0$-line yields the follow Jacobian matrix $\theta(n_i^*, V_i^*)(i=1 \text{ or } 2)$.

$$\theta(n_i^*, V_i^*) = \begin{pmatrix} \frac{a_R \frac{d\omega(n_i^*)}{dn}}{(H\bar{A}\omega(n_i^*)+L)^2} & -1 \\ -\frac{J}{(\rho n + \frac{L}{a_R \omega(n_i^*)})^2}\frac{1}{\omega(n_i^*)}\left[\rho\omega(n_i^*) - \frac{L}{a_R}\frac{\alpha}{(\sigma-1)(1-\alpha)}\frac{1}{n_i^*}\right] & -1 \end{pmatrix}$$

Moreover, we obtain the determinant of this matrix using a fact that both $\dot{n}=0$ and $\dot{V}=0$ are satisfied in the steady states. The determinant is given by

$$\det\theta(n_i^*, V_i^*) = \frac{(V_i^*)^2}{Ja_R}\left\{(1-J)\left[\frac{\alpha}{(\sigma-1)(1-\alpha)}\frac{L}{\omega(n_i^*)n_i^*}\right] - \rho a_R\right\}. \tag{37}$$

Because the slope of $\frac{(1-J)L}{\omega(n)}$ is greater than ρa_R, we derive

$$\rho a_R > (1-J)\frac{\alpha}{(\sigma-1)(1-\alpha)}\frac{L}{\omega(n_2^*)n_2^*}, \tag{38}$$

which implies that the determinant is negative at the upper steady state, that is det $\theta(n_2^*, V_2^*)$. On the other hand, det $\theta(n_1^*, V_1^*)>0$ at the lower steady state.

By using the determinant of the Jacobian, we can derive the change of n as

$$\frac{\partial n_i^*}{\partial H} = -\frac{1}{\det\theta(n_i^*, V_i^*)}\left[\frac{\bar{A}a_R\omega(n_i^*)}{(H\bar{A}\omega(n_i^*)+L)^2}\right], \tag{39}$$

which implies that $\partial n_2^*/\partial H$ is positive at the upper steady state and it is negative at the lower steady state.

The change of steady state variety is expressed by Figure 6(a).

[6] However $V=0$-line below $n=0$-line is also steady state, we pay little attention to this region because we are interested in the change of variety.

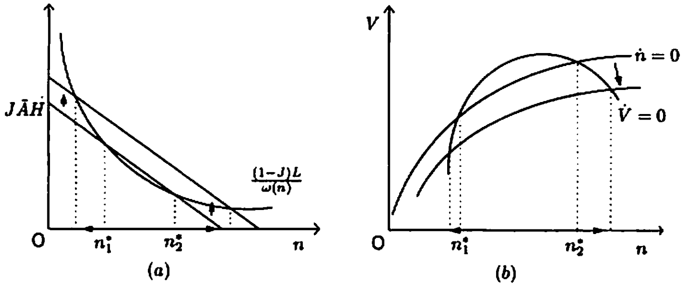

Figure6

Figure 6(a) shows that increase in human capital H lift up the straight line. Consequently, the variety of lower steady state decreases and that of upper steady state increases. On the other hand, the change of phase diagram is illustrated in Figure 6(b). We observed the increase in H shifts $\dot{n}=0$-line down and $\dot{V}=0$-curve does not move because it is independent of human capital. Therefore we easily confirm that the variety of upper steady state rises and that of lower steady state goes down.

In this analysis, the production of good 2 decreases when the economy reaches the new upper steady state. This phenomenon results from the follow reasons. First reason is due to usual Rybczynski Theorem. The increase of H raises the production of good 1 which is more human capital-intensive than that of good 2. Hence, Y_2 goes down. Second one is due to a fact that the rise of variety provides an increase of relative wage between human capital and labor. Remember that ω is a increasing function of n. This implies that expanding variety reduces the wage of labor and supplier of good 1 chooses more labor(equivalent to intermediate goods)-intensive technology. Consequently, more quantity of labor are employed for the production of good 1 through intermediate goods sector.

Proposition 1 *When the endowment of human capital increases, the variety of intermediate goods in the upper steady state rises and that in the lower steady state declines. Moreover, in the upper steady state, the production of good 1* *increases and that of good 2 decreases.*

Next we consider a expansion of labor L. The change of variety through the increase of labor is given by

$$\frac{\partial n_i^*}{\partial L}=\frac{1}{\det\theta(n_i^*,V_i^*)}\frac{(V_i^*)^2(1-J)}{Ja_R\omega(n_i^*)},\tag{40}$$

which implies that $\frac{\partial n_1^*}{\partial L}$ >0 at lower steady state and $\frac{\partial n_2^*}{\partial L}$ >0 at upper steady state. This case can be analyzed as same as the case of human capital. In Figure 7(a), The increase of labor L shifts the curve which corresponds to $\frac{(1-J)L}{\omega(n)}$ upward. Therefore, the variety of upper steady state goes down and that of lower steady state goes up.

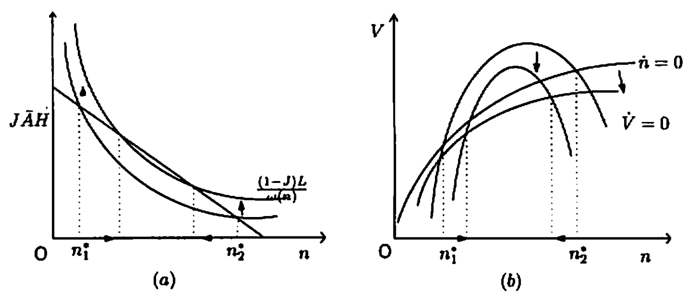

Figure7

The change of phase diagram is depicted in Figure 7(b). The increase of L also shifts $\dot{n}=0$ downward. On the other hand, it shifts $\dot{V}=0$-line downward too. In this case, the value of V at upper steady state declines and that at lower steady state rises. However we cannot estimate the change of variety from the phase diagram only, we are able to confirm that the variety of upper steady state declines and that of lower steady state rises from above analysis.

At upper steady state, the production of good 1 rises and that of good 2 declines with the increase of labor L as same as the case of human capital. This conclusion also results from two effects. One is the Rybczynski effect ; the production of good 2 increases. Moreover, suppliers of good 1 choose more human capital intensive technology because the variety of upper steady state decreases. As a result, employed labor in the intermediate goods sector declines and the production of good 1 rises. Therefore, the production of good 1 decreases and that of good 2 increases at upper steady state.

Proposition 2 *When the endowment of labor increases, the variety of intermediate goods in the upper steady state declines and that in the lower steady state rises. Moreover, in the upper steady state, the production of good 1 decreases and that of good 2 increases.*

Next we consider the change of relative price q_1/q_2. For convenience, we define $q = q_1/q_2$. In this case, increase in q, which implies that the price of good 1 goes up relatively, provides the higher wage of human capital[7]. Therefore, $\frac{\partial \omega}{\partial q}>0$ and it is a natural conclusion of Stolper-Samuelson Theorem. The change of variety due to the rise of q is given by

$$\frac{\partial n_i^*}{\partial L} = \frac{1}{\det \theta(n_i^*, V_i^*)} \frac{(V_i^*)^2(1-J)}{Ja_R\omega(n_i^*)}, \tag{41}$$

which says that $\frac{\partial n_i^*}{\partial q}$ >0 at the lower steady state and $\frac{\partial n_i^*}{\partial q}$ >0 at the upper steady state.

This case is illustrated in Figure 8. Figure 8(a) shows that the increase in q shifts the gragh of $\frac{(1-J)L}{\omega}$ down because ω is an increasing function of q. Hence, we easily understand that the variety of upper steady state goes up and that of lower steady state goes down.

[7] Because the wage of labor is fixed at $w = q_2$, w_H absolutely increases.

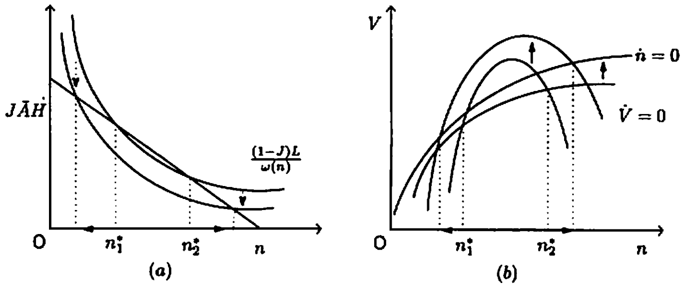

Figure8

Figure 8(b) shows the change of phase diagram. In this case, both $\dot{n}=0$-curve and $\dot{V}=0$-curve shift upward. However we cannot confirm that the direction of change when we observe the shifts of two curves, we know that upper steady state moves to upper right and lower steady state moves to lower left.

Moreover, we can derive the change of production at steady states more easily because only change of the variety influences. First, we consider the upper steady state. The rise of n_1^* makes the suppliers of good 1 choose more intermediate goods-intensive technology. Because all of human capital is devoted to the production of good 1, demand for intermediate goods increases. As a result, more intermediate goods are manufactured and less labor is used for good 2. From these analysis, we can conclude that the production of good 1 rises and that of good 2 reduces at the upper steady state. Moreover, we can also say that the production of good 1 decreases and that of good 2 increases at the lower steady state by using same procedure.

Proposition 3 *When the relative price of consumption goods q rises, the variety of intermediate goods in upper steady state increases and that in lower steady state decreases. Moreover, in the upper steady state, the production of good 1 rises and that of good 2 declines. On the other hand, in the lower steady state, the production of good 1 decreases and that of good 2 increases.*

V Concluding Remarks

In this paper, we have established the growth model of a small open economy that includes the intermediate goods market under monopolistic competition. We introduce two production factors, those are human capital and labor. We have assumed that good 1 is intermediate goods-intensive and good 2 is labor-intensive. To consider the influence of two intensities is main contribution of this paper. From those characteristics, increase of variety provides the difference of productivity in the consumption good sectors and we can observe the change of relative factor price.

In the section 3, we investigated the dynamic behavior of the model and we found the possibility that there exist two steady states. When these upper and lower steady states exist, it is possible that the economy converges to both the steady states even if the economy starts with particular initial value. Which steady state is chosen depends on people's expectation. We showed that the equilibrium path is not uniquely determined because of the multiplicity of steady states. In the section 4, we tried comparative statics of steady states. Especially, we consider how human capital, labor, and terms of trade affect the steady states. Multiplicity of steady states provides contrary conclusions of comparative statics between upper and lower steady states. For example, the increase in human capital raises the variety of upper steady state. However, that declines the variety of lower steady state.

Our results suggest some future researches. In particular, it seems to be important to establish a growth model that can provide sustainable growth by introducing accumulation of human capital. In this paper, the variety of intermediate goods became constant at steady states. This implies that the economy eventually stops its growth process. Therefore, from a viewpoint of new growth theory, it may be interesting to consider the problem of long-run growth by introducing human capital accumulation. In addition, we considered the implications of some economic policies by doing comparative statics. But the effects of economic policies should be investigated from a view of welfare analysis. Therefore, the consideration of economic policies in this paper may be incomplete. As a result, to analyze optimal policies is an important task for our future research.

Appendix : Derivation of Dynamical System

In this subsection, we attempt to reduce the model to the system of two differential equations by using the above analysis. At the begining, we express the production function of good 1 with $u_H(t)H$. From (23),

$$Y_1(t) = \left(\frac{q_1}{q_2}\right)^{-1} \frac{1}{1-\alpha}\omega(n(t))u_H(t)H. \tag{42}$$

Therefore, by combining with (19), total amounts of profit derived by intermediate goods sector is given by

$$n(t)\pi(t) = q_2\frac{\alpha}{\sigma}\frac{1}{1-\alpha}\omega(n(t))(H - a_R\dot{n}(t)). \tag{43}$$

Hence, by using (43) and (24), the behavior of $\dot{n}(t)$ is obtained after some calculation, that is

$$\dot{n}(t) = \frac{H}{a_R} + \frac{L}{\bar{A}a_R\omega(n(t))} - \frac{E(t)}{\bar{A}a_R w_H(t)}. \tag{44}$$

If $\dot{n}(t) > 0$, $w_H a_R = v(t)$ is satisfied. Here we define a new variable $V(t)$ as $V(t) = \frac{v(t)}{E(t)}$, then the behavior of $n(t)$ is represented by

$$\dot{n}(t) = \max\left\{\frac{H}{a_R} + \frac{L}{\bar{A}a_R\omega(n(t))} - \frac{1}{\bar{A}V(t)}, 0\right\}.$$

Moreover, a growth rate of $\dot{V}(t)$ is derived as $\dot{V}(t)/V(t) = \dot{v}(t)/v(t) - \dot{E}(t)/E(t)$. From (3),

$$\dot{V}(t) = \rho V(t) - \frac{\pi(t)}{E(t)}. \tag{45}$$

(17), (42) and $q_1 Y_1(t) + q_2 Y_2(t) = E(t)$ yield

$$\frac{\pi(t)}{E(t)} = \frac{\frac{1}{n(t)}\frac{\alpha}{\sigma}\frac{1}{1-\alpha}\omega(n(t))}{\frac{1}{1-\alpha}\omega(n(t)) + \frac{u_L(t)L}{u_H(t)H}}. \tag{46}$$

On the other hand, from (23), labor-human capital ratio is given by

$$\frac{u_L(t)L}{u_H(t)H} = \frac{L - \frac{\alpha}{1-\alpha}\left(1-\frac{1}{\sigma}\right)\omega(n(t))u_H(t)H}{u_H(t)H}. \tag{47}$$

Moreover, $u_H(t)H$ is expressed by the differential equation for $n(t)$, that is

$$u_H(t)H = H - a_R\dot{n}(t) = -\frac{L}{\tilde{A}\omega(n(t))} + \frac{a_R}{\tilde{A}V(t)}. \tag{48}$$

Therefore,

$$\frac{u_L(t)L}{u_H(t)H} = \frac{L\tilde{A}V(t)\omega(n(t))}{a_R\omega(n(t)) - LV(t)} - \frac{\alpha}{1-\alpha}\left(1-\frac{1}{\sigma}\right)\omega(n(t)). \tag{49}$$

Hence, $\frac{\pi(t)}{E(t)}$ is represented by

$$\frac{\pi(t)}{E(t)} = \frac{1}{n(t)}J\left(1 - \frac{LV}{a_R\omega(n(t))}\right). \tag{50}$$

As a consequence, the behavior of $V(t)$ is derived as the following relation, that is,

$$\dot{V}(t) = \rho V(t) - \frac{1}{n(t)}J\left(1 - \frac{LV}{a_R\omega(n(t))}\right).$$

Finally, the following condition must be satisfied on the equilibrium path from (27) and (26), that is,

$$\lim_{t\to\infty} V(t)n(t)e^{-\rho t} = 0.$$

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